# A Procedure for Decomposing the Myoelectric Signal Into Its Constituent Action Potentials—Part II: Execution and Test for Accuracy

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***Abstract*—The scheme for decomposing a myoelectric signal into its constituent motor unit action potential trains described in the paper [3] requires interaction from the human operator. In this paper, guidelines to be employed by the operator in assisting the computerized algorithms in identifying (classifying) a motor unit action potential are presented. The accuracy of the decomposition scheme was evaluated by decomposing a mathematically synthesized myoelectric signal. This signal was constructed by linearly superimposing eight mathematically generated motor unit action potential trains along with Gaussian noise. A skilled operator was able to decompose this signal with an accuracy of 99.8 percent, incurring one error in a total of 435 classifications. The decomposition reproducibility was evaluated by having two experienced operators independently decompose the same record of empirically obtained myoelectric signal. Their results were in total agreement for 479 motor unit action potential classifications belonging to five motor unit action potential trains. Up to eight motor unit action potential trains have been decomposed from one myoelectric signal.**

## INTRODUCTION

THE signal recording, processing and computer-assisted decomposition technique described in the preceding paper (LeFever and De Luca [3]) was used to study the motor unit (MU) firing properties of the first dorsal interosseous and deltoid muscles.

This paper will discuss empirically observed properties of MU firing behavior which can be expoited to obtain a highly accurate analysis of a myoelectric (ME) signal when there is substantial superposition and instability of the motor unit action potential (MUAP) waveforms. Secondly, this paper will show that it is possible to obtain a highly accurate record of the firing times of several concurrently active MU's when the operator of the previously described computerized system [3] makes decisions consistent with these observations.

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It is impossible to measure the accuracy of the decomposition program with real ME signals in an absolute sense. The true occurrence times of all the MUAP's in the ME signals are not known; therefore it is not possible to state absolutely when the decomposed MUAP's have been correctly identified. This limitation has been circumvented by proceeding with two alternative approaches. First, synthetic ME signal data were generated and the computer program was used to decompose them. This approach has the advantage of having precise knowledge of all the MUAP's that comprise the synthetic signal so that an objective evaluation of the accuracy of the decomposition procedure may be made. Second, two experienced individuals independently decomposed an identical record of real ME signal, and their results were compared. The combined results of these two approaches were taken as an indication of the accuracy of the decomposition procedure.

## EMPIRICALLY DETERMINED PRESUMPTIONS USED IN THE DETECTION OF MOTOR UNIT ACTION POTENTIALS

The decomposition program basically employs two criteria for identifying a MUAP; its detected waveform shape and the fact that the interpulse interval (IPI) of a motor unit action potential train (MUAPT) may be modeled as a renewal process with Gaussian distributed intervals. See an accompanying paper by LeFever and De Luca [3]. However, it was found that these two properties are not always sufficient for correctly identifying a MUAP within an ME signal. When the decomposition program is unable to arrive at a decision, it requests assistance from the operator. The operator then employs a series of essential presumptions which further describe the firing behavior of MUAP's.

The following essential presumptions have been determined by close scrutiny of numerous MUAPT's that could be easily identified visually, by virtue of their comparatively large and distinctive shapes.

1) The firings and resulting IPI's of each MU can be modeled by a rapidly varying, quasideterministic input (mean firing rate) to a stochastic event generator.

2) The input to each MU is substantially correlated to other MU's present within the ME signal and to the measured force of the muscle contraction.

3) MUAP waveforms produced by several fibers from the same MU may vary considerably (mean-squared difference >20

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percent) from one firing to the next. These MUAP waveforms will be, in general, multiphasic (greater than three). This degree of variation is often too large to permit automatic detections to be made by the decomposition program.

The simultaneous display of the force generated by the muscle and the IPI's or firing rates of concurrently active MU's has made possible the observations which led to the first two presumptions. These observations may be summarized as follows:

a) In MUAPT's with quasiregularly occurring (coefficient of variation less than 20 percent), distinctive MUAP waveforms, IPI's more than twice or less than half the local mean (referred to as "double" of "half" IPI's), were not observed when there was no sharp change in the force, and when no similar intervals were present in firings of the other detected MU's.

b) When double or half IPI's were observed for regularly discharging MU's, they were observed in all detected MU's and an abrupt change in force occurred at the same time.

c) The mean firing rates of concurrently detected MU's have been observed to modulate (rapidly vary about a more slowly varying mean) with a high degree of correlation (as high as 0.91). This can often be observed during the decomposition at the single interval level. That is, at any point in time, the relative IPI's (each normalized by its mean value) of each of the detected MU's tend to cluster either all above, near, or below unity.

d) When the decomposition has resulted in a double or half interval for only one of the detected MU's at any given time, a corresponding transient deflection appears in the plot of its mean firing rate, as compared to the firing rates of the other MU's. A "correction" to this error (determined by examining that particular section of the ME signal to find a missed or false detection) results in the elimination of the transient deflection.

The observations which support the third presumption are as follows:

a) Stålberg et al. [5] have reported that the mean value of consecutive interpotential-interval differences (between two fibers of the same MU) can range from 2 to 70 $\mu$s.

b) Several of the ME signals analyzed in this study have contained two distinct, nonoverlapping waveforms which always occurred within a stable interval (standard deviation of 20 to 50 $\mu$s) of each other.

c) The statistics of the intervals between highly variable, multiphasic yet quite distinctive waveforms (as compared to all other waveforms in the ME signal), are consistent with those found for the more stable biphasic MUAP waveforms.

d) The variability in time between individual peaks within these multiphasic waveforms is consistent with a) and b) above. This phenomenon (illustrated in Fig. 1) is often apparent in the ME signals obtained in our studies and most likely results from the following two factors. The high fidelity recording technique and high-pass filtering results in ME signals similar to the single fiber recordings (using single fiber electrodes) reported by Ekstedt and Stålberg [2] and others. The larger area of the recording electrode, however, results in the detection of a greater number of fibers, several of which may belong to the same MU.

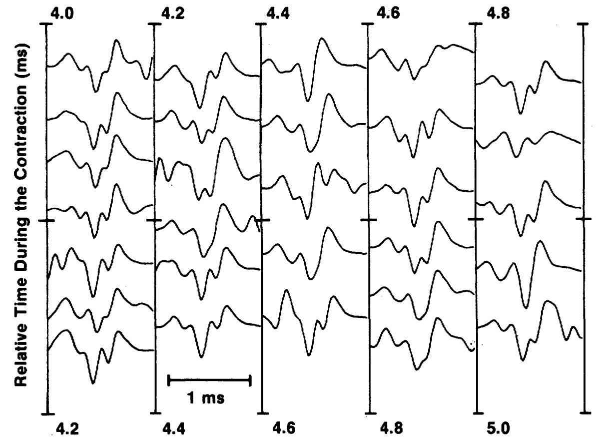


Fig. 1. Consecutive discharges of a motor unit recorded with the electrode described in the previous accompanying paper. The record begins at the top of the left-hand column, proceeds vertically and is column-wise continuous. The vertical displacement between motor unit action potentials corresponds to the interpulse interval duration. Note the substantial and erratic variability that is present in the waveform. This behavior is typically observed when the motor unit action potential waveform consists of action potentials from only a few (probably less than five) muscle fibers in the vicinity of the electrode recording contacts. The random-varying appearance is most likely the result of the randomly varying discharge times (end-plate jitter) of different muscle fibers within the same motor unit [2].

## Implementation of the Empirical Presumptions

The implementation of the essential presumptions in the decomposition procedure may be divided into the following three steps:

*Step 1:* Scan through the ME signal. The main purpose of this initial scan is to determine the number and shape of the different MUAP waveforms present, approximately where each first occurs (time of recruitment), the amount of change in the MUAP waveforms throughout the contraction, the regularity with which each MU fires, and if digital filtering may be useful.

*Step 2:* Perform a tentative decomposition of the ME signal. This is the most time consuming portion of the analysis. Some of the more difficult ME signal records, 18 to 32 s long, have required over ten continuous hours to detect up to 3000 MUAP's from the ME signal. During the decomposition, the operator has available a plot (hard copy) of the subject's force during the contraction. This is used to help him make decisions about the IPI statistics since abrupt changes in the force are preceded by abrupt changes in the IPI's. The scan performed in the previous step usually permits the operator to identify a MUAP waveform as a newly recruited MU when it first occurs.

*Step 3:* Plot the IPI and firing rates of the detected MU's. The IPI plots of each MU are scanned for any "long" or "short" IPI's, relative to its adjacent IPI's, and IPI's of other MU's. The firing rate plots are also scanned for transient deflections in the firing rate of one MU as compared to that of the other MU's. Transient deflections in the firing rate and abnormal IPI's usually occur simultaneously. The decomposition program is then again used to repeat the analysis of the section(s) of the ME signal at the time(s) of these anomalies. If any corrections can be justified according to the previously described criteria, these corrections are made to the firing time data file. If the corrected IPI and firing rate plots display a more consistent behavior of the MU's, these corrections are assumed to be valid. Any corrections which do not increase the consistency of the results are retracted. When all corrections which can be justified are made, the decomposition is done. These types of corrections are usually only required for less than 5 percent of the detections of the lower amplitude and/or less distinctive MUAP waveforms. The analysis is therefore mildly biased toward obtaining consistent firing statistics for all the MU's detected.

The total time which has been required to obtain the final decomposition of a compressed and filtered ME signal is dependent upon the number of MUAPT's detected, and the number of operator decisions required. The decomposition time, as a function of these two factors, grows more rapidly than linear. The stability and distinctiveness of each MUAP waveform and the degree of superposition significantly influence the amount of operator interaction required. Extraction of only the one or two MUAPT's with the largest amplitude MUAP waveforms usually proceeds very rapidly, because most of the interactive features of the decomposition procedure are either not required or seldom used. If an attempt is made to extract every MUAPT that can be reliably detected, the time required to perform the decompositions ranges from 5 to 30 min/s of ME signal.

## Evaluation of the Decomposition Accuracy

The final presumption made about the detection of MU firings is that a "reliable" decomposition of an ME signal into its constituent MUAPT's is an accurate account of the activity of those MU's. To be reliable, the decomposition must be reproducible, and should not contain any sections in which the firing statistics of an MU are inconsistent with its own firing statistics (in other portions of the contractions), or the firing statistics of the other detected MU's. None of the decompositions performed thus far, in which all MUAP waveforms were distinct and all superpositions were unquestionably resolved, exhibited sections with grossly inconsistent firing statistics.

To support the assumption that reliability implies accuracy, a study was made to evaluate the accuracy with which a synthetic ME signal could be decomposed, and the reproducibility of the decomposition of an actual ME signal. The purpose of the study with synthetic data was to determine if a reliable decomposition of a known signal, similar to an actual ME signal, was indeed accurate. The purpose of the reproducibility study was to determine if a decomposition judged reliable by one individual could be reproduced independently by another individual. The two individuals most experienced in the use of the decomposition technique participated in this study. These two individuals each had spent at least 400 h decomposing ME signals using the computer program. In addition, a third individual with considerably less experience (novice operator) also participated. The novice operator was included to evaluate the effect of experience on the accuracy and reproducibility. Prior to decomposing the synthetic and real ME signals used for this study, the novice operator had spent approximately 10 h observing the two skilled operators perform other decompositions. He had also practiced decomposing other ME signals, with guidance, for approximately 6 h. An additional 5 h were spent decomposing the real ME signal and then five more hours decomposing the synthetic ME signal.

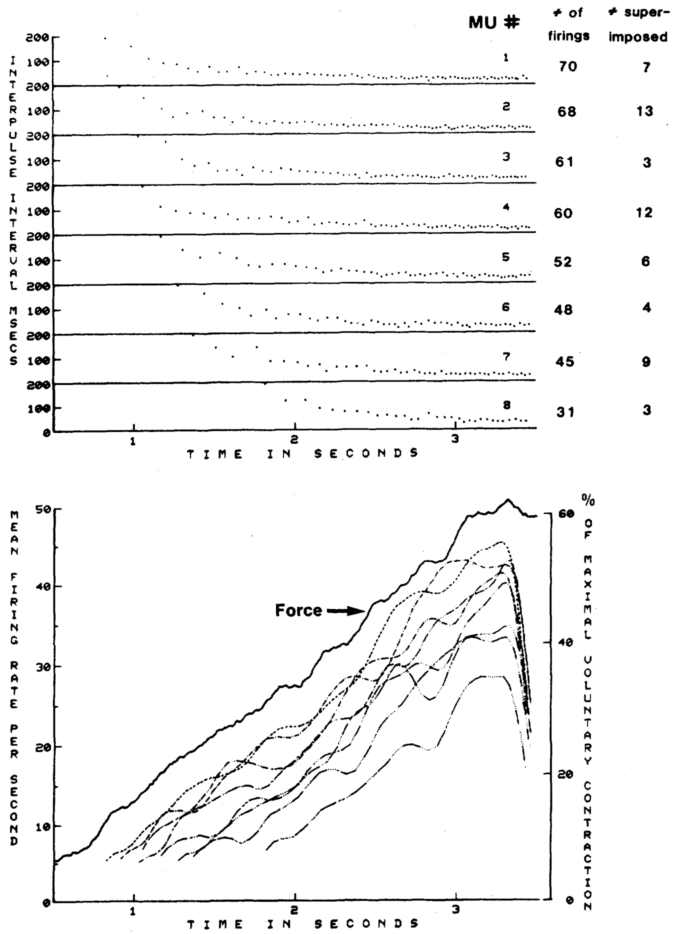


Fig. 2. The top plot contains the actual interpulse intervals of the motor unit action potential trains that were superimposed to form the synthetic myoelectric signal. The second column on the right displays the number of occurrences in each train, and the third column displays the number of occurrences in each train which overlapped with occurrences in one of the other trains. The bottom plot contains the corresponding firing rates of the eight motor unit action potential trains along with the force that was used to generate the synthetic firing times. The apparent decrease in the firing rate at the end of the record is due to the step response of the firing rate smoothing filter which consisted of a Hanning window having a width of 400 ms.

### *Method for Producing a Synthetic Myoelectric Signal*

To be able to objectively measure the accuracy of a decomposition, it was necessary to create a synthetic ME signal, for which the firing times of each MU are known. A program was written (LeFever [4]) which simultaneously generated several MUAPT's and summed them to create the synthetic ME signal.

The firing times of each MUAPT were created by using the force produced by a subject during an actual contraction, as the common input to a stochastic event generator. Prior to the simulation, a force threshold for recruitment was randomly generated for each MU. At any point in time throughout the simulated contraction, the mean firing rate of an MU was proportional to the difference between the force input (to the stochastic event generator) and the recruitment threshold. Thus, the firing statistics conformed to one of the empirical assumptions. The stochastic event generator used a renewal process to create each successive firing time. For a constant force input, Gaussian distributed intervals with a standard deviation equal to 15 percent of the mean interval would be generated.

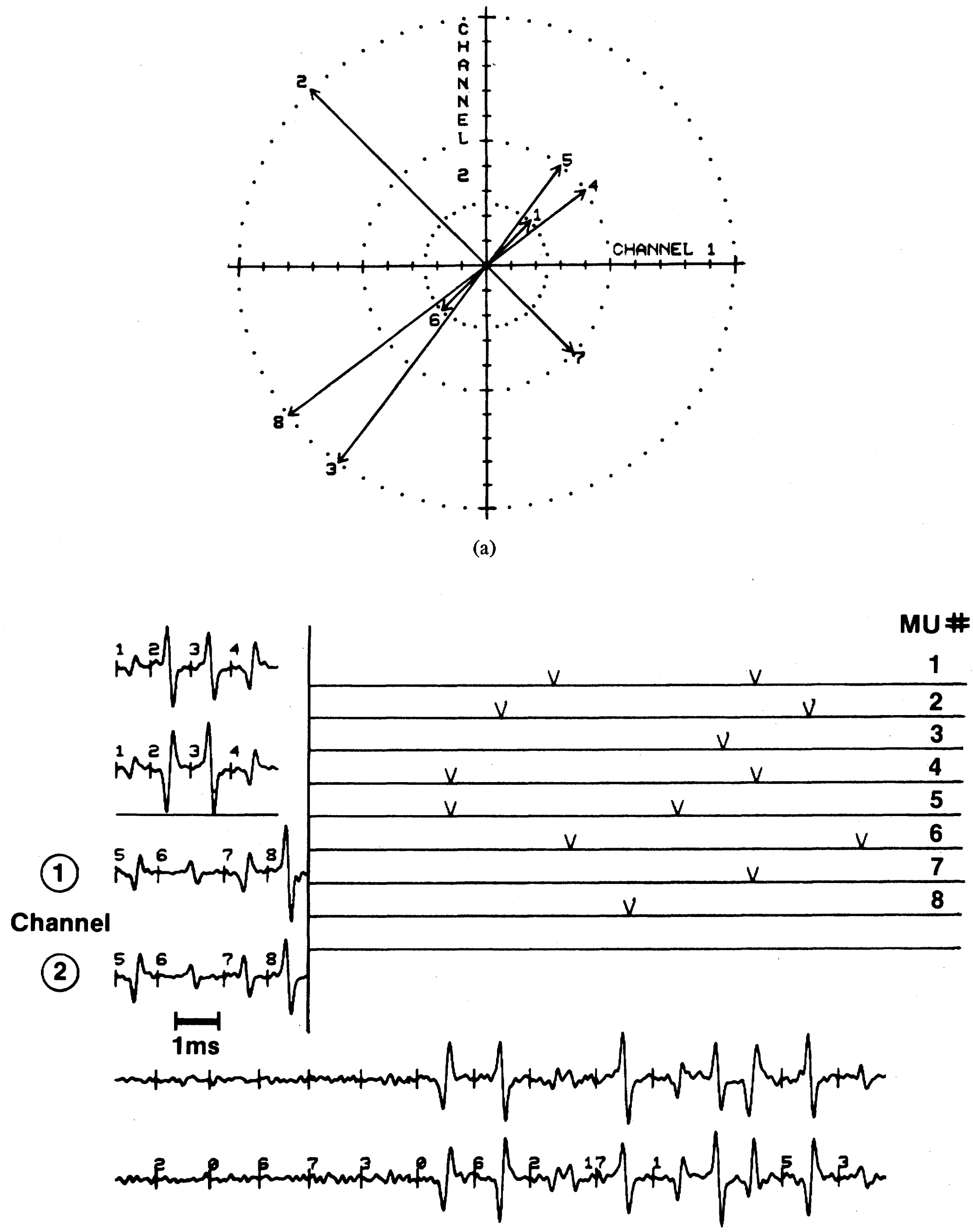


Fig. 3. (a) Signal space representation of the synthetic motor unit action potential waveforms. Each vector shows the relative amplitude on each of the two channels of each motor unit's action potential waveform. (b) A display of the decomposition of the synthetic myoelectric signal. Two channels of the compressed synthetic signal are displayed on the bottom. The templates used by the decomposition program are presented on the left hand border. The occurrences of the action potentials of each of the eight motor units is marked with a *V*.

The firing rates and IPI's of the eight synthetic MUAPT's used in this study and the force used to generate them are shown in Fig. 2. Of the total 435 firings generated, 57 resulted in the superposition of MUAP waveforms. The number of firings of each MU and the number of superpositions it was involved in are noted in Fig. 2. Synthetic data were generated to simulate a contraction of 3.5 s duration.

The MUAP waveform of each MU was fixed throughout the simulated signal and was characterized by an impulse response. The $i$th MUAP waveform (impulse response) on the $j$th channel $s(t)$ was expressed by the following formula:

$$s_{ij}(t) = A_{ij}h(t) = A_{ij}\left(\frac{t}{(b \cdot t)^4 + c}\right); \qquad -\infty < t < \infty.$$

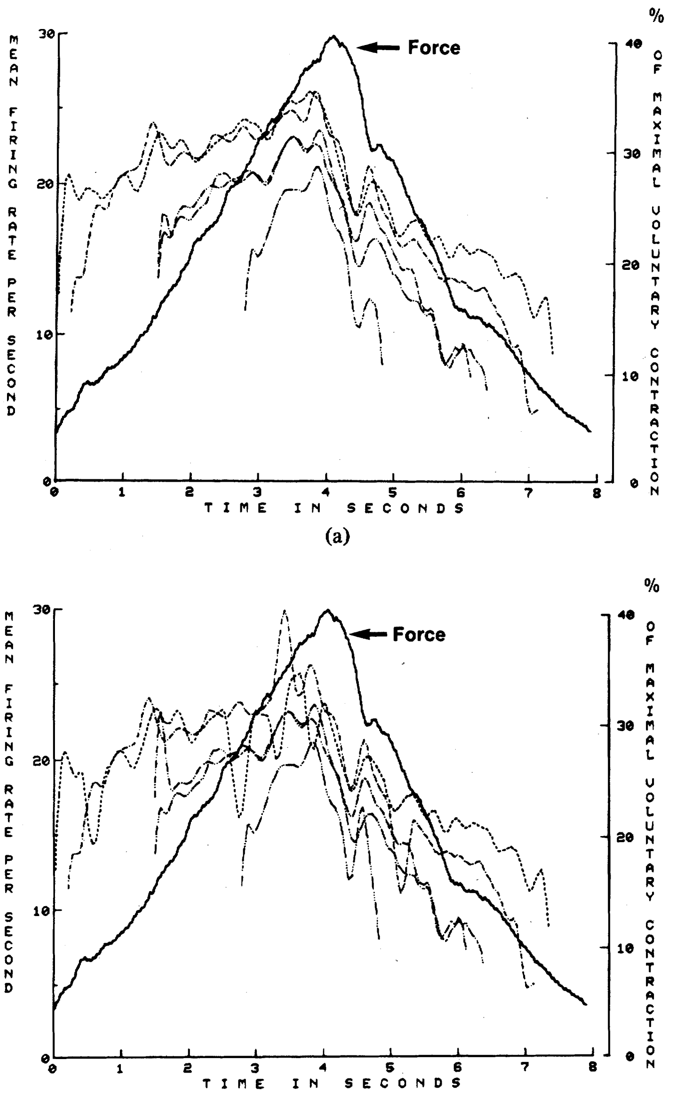


Fig. 4. (a) The firing rates of five motor units decomposed from a real myoelectric signal by the two experienced individuals along with the force tracing of the isometric contraction that produced the myoelectric signal. (b) The firing rates of the same five motor units decomposed by the novice individual. These firing rates were obtained from a decomposition which had 2.5 percent incorrect classifications.

The values of $b$ and $c$ were the same for all MUAP waveforms. This permits a very direct two-dimensional signal space representation of each MUAP by specifying the value of the amplitude $A_{ij}$ for each of the two channels. The signal space representation for the eight MUAP waveforms contained in the synthetic ME signal is shown in Fig. 3(a). The lower portion of Fig. 3(b) displays a section of the compressed two channels of the synthesized ME signal. The waveforms of the eight MUAPS (two channels for each) appear on the upper left hand border. The remainder of the figure displays the time of occurrence of the detected MUAP's.

A zero mean, Gaussian random variable was added to each sample of the synthetic ME signal to simulate noise. The value of the noise standard deviation used was 40 percent of the peak amplitude of the smallest MUAP waveform.

### *Results of the Synthetic Myoelectric Signal Decomposition*

The synthetic ME signal was decomposed independently by one of the skilled operators and by the novice operator. Both operators were told that the synthetic ME signal was intended to simulate the characteristics of real ME signals. They were not informed that there were eight MUAPT's present, nor that the MUAP waveforms would remain stable. The decomposition performed by the skilled operator contained only one crossover error; all other detections for all eight MUs were accurate. The crossover error occurred when a firing of MU 4 was detected as MU 5 and the next firing of MU 5 was detected as MU 4. This error occurred when MU's 4 and 5 fired within 1 ms of each other, and were both superimposed with larger amplitude MUAP waveforms. The smallest signal space distance, among the eight MUAP waveforms, occurs between these two MU's. Thus, a crossover between these two is most likely. Since this error resulted in only a 1 ms error in the detected firing times of MU's 4 and 5, no discernible difference could be found between the true IPI and firing rate plots, and those for this decomposition. The time required to obtain this decomposition was approximately 5 h. This is longer than would be required for the same time-duration of a real ME signal; however, the synthetic ME had more MUAPT's with more superpositions than would normally be encountered in a typical, empirically obtained, ME signal.

The novice operator was able to detect only six of the eight MUAPT's. He ignored the two lowest amplitude MUAP waveforms. Four of the remaining six MUAPT's were detected without error. The detection errors of the remaining two, MU's 4 and 5, were mostly caused by confusion between their similar MUAP waveforms. Firings of MU 4 were detected as firings of MU 5, on five occasions. Firings of MU 5 were twice detected as firings of MU 4, skipped twice and falsely detected once. Two crossovers were represented by these errors, one the same crossover error made by the skilled operator.

## Evaluation of the Decomposition Reproducibility

For this portion of the study, one of the skilled operators selected an ME signal record for which he believed his decomposition to be "reliable" throughout the contraction. A decomposition of the same ME signal was then performed by the other two individuals. Neither was given information about the decompositions performed by anyone else, prior to or during the decomposition procedure.

The ME signal selected contained five MUAPT's which the skilled operator believed had been reliably detected. The force output and firing rates of these five MU's are shown in Fig. 4(a). The firing rates of the synthetic data can be seen to fluctuate more randomly than the firing rates obtained from the real ME signal. (Compare Fig. 4(a) with Fig. 2.) This implies that the randomness of the real firing rates is considerably less than the 15 percent randomness introduced into the synthetic data. The interval statistics of the synthetic ME signal are therefore of less help in performing a reliable decomposition than the interval statistics of the real ME signal. The MUAP waveforms of MU's 4 and 5 in the real data were larger in amplitude than the other three, but underwent large changes during the contraction

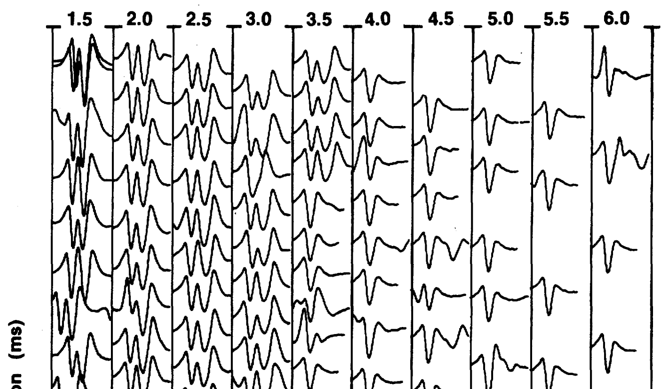


Fig. 5. A raster plot of two of the motor unit action potential trains whose mean firing rate is presented in Fig. 4. The record begins at the top of the left-hand column, proceeds vertically and is column-wise continuous. The vertical displacement between the motor unit action potentials corresponds to the interpulse interval duration. Note the gradual but substantial variation in the waveform. This behavior is most likely due to the movement of the recording electrode with respect to the active muscle fibers. (The occasional drastic changes in waveshape are the result of superpositions with other motor unit action potentials.)

as illustrated in Fig. 5. This figure presents rasters of all the MUAP's within the two motor unit trains. The first occurrence appears on the top left hand border; the subsequent occurrences progress downwards and are column-wise continuous.

The results of the decompositions of the real ME signal data are as follows: both skilled operators were 100 percent in agreement for the detections of a total of 479 MUAP's from five MU's. The performance of the novice operator contained a total of 12 errors, as detailed in the accompanying table, yielding an accuracy of 97.5 percent. An offset error (refer to Table I) is a combination of a missed and false detection between two correct detections in an MUAPT. The effect of an offset error on the mean firing rate is small, since the number of firings is not changed. Isolated missed or false detections, however, produce pronounced transient deflections in the mean firing rate, as illustrated in Fig. 4(b) which presents the firing rate plots of the decomposition performed by the novice operator.

TABLE I
PERFORMANCE OF NOVICE OPERATOR COMPARED TO THE IDENTICAL RESULTS OF BOTH EXPERIENCED OPERATORS

| MU | TOTAL # OF MUAPS | MISSED DETECTIONS | FALSE DETECTIONS | OFFSET ERRORS |
|---|---|---|---|---|
| 1 | 146 | 5 | 0 | 2 |
| 2 | 132 | 1 | 1 | 1 |
| 3 | 83 | 0 | 1 | 0 |
| 4 | 83 | 0 | 0 | 0 |
| 5 | 35 | 0 | 1 | 0 |

## SUMMARY

The technique described in the previous accompanying paper (LeFever and De Luca [3]) and evaluated in this paper reliably separates motor unit action potentials (MUAP's) from a myoelectric (ME) signal. Up to eight motor unit action potential trains (MUAPT's) have been decomposed from one ME signal. The time required to perform the computer assisted decomposition of a signal is dependent on many factors; the two major ones being the quality of the ME signal and the number of MUAPT's to be extracted. In the case where eight MUAPT's are to be extracted from a signal recording during contractions greater than 60 percent of maximal, the time required ranges from 5 to 30 min/s of signal. However, if only two or three MUAPT's are to be extracted from a signal recorded during contractions less than 30 percent of maximal, the time required can be as little as 1 min/s of data.

The reliability of the decomposition procedure was tested by having a sample ME signal containing five MUAPT's independently decomposed by two experienced individuals, each with more than 400 h of experience. Their results were 100 percent in agreement. One of the experienced individuals performed a decomposition on a mathematically synthesized ME signal containing eight MUAPT's, having a total of 435 MUAP's; 57 of which were superimposed. His results were accurate except for one insignificant error, which had essentially no effect on the firing statistics. The decompositions of the same two ME signals by a less experienced person contained errors in the detection of low (relative) amplitude MUAP waveforms and between MUAP waveforms that were closest in signal space.

These results support the assumption that a reliable decomposition performed by a highly experienced person, using all of the methodology previously described, is an accurate decomposition. These results also demonstrate that an operator requires significantly more than 20 h experience to be able to obtain a completely reliable decomposition of complex ME signals. Furthermore, a 2.5 percent error rate in the classification of MUAP's results in substantial discrepancies in the behavior of the calculated firing rates of the MU's, as may be seen in Fig. 4(a), (b).

## ACKNOWLEDGMENT

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**Ronald S. LeFever** (S'69-M'70), for a photograph and biography, see this issue, p. 157.

**Alan P. Xenakis**, photograph and biography not available at the time of publication.

**Carlo J. De Luca** (S'64-M'72-SM'77), for a photograph and biography, see this issue, p. 157.


# Sensitivity Analysis and Improved Identification of a Systemic Arterial Model

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***Abstract*—An identification scheme for the determination of several parameters associated with a third-order lumped-parameter model of the systemic arterial system has been developed previously [1]. In the present study a parameter sensitivity analysis of this model is conducted in conjunction with the identification scheme. This analysis indicates that the state variables of this system are indeed sensitive to changes in the model parameters and initial conditions, but not unduly so. It also indicates the relative sensitivities of individual model parameters and provides valuable insight into the behavior of the arterial model. This insight has resulted in a significant simplification and modification of our identification scheme [1] that makes it more likely to be employed in practice.**

**The modified identification scheme may be employed with either one or two arterial pressure measurements and a cardiac output measurement specified as input to parameter estimation process. The method is more accurate, however, when both a proximal (aortic root) pressure and a distal (brachial artery) pressure are utilized as inputs. This study shows that the modified scheme offers the attractive alternative of providing a reasonably accurate prediction of aortic root pressure using as input a single distal arterial pressure measurement and a cardiac output measurement. This feature should be of considerable interest in the clinical setting.**

**The arterial model mentioned above is driven by aortic root flow $f_a(t)$ and its time derivative $(\dot{f}_a(t))$, and an adequate specification of this forcing waveform is quite important to the accuracy of the identification process. In this study we are concerned with the feasibility of this identification method as employed in clinical practice where, usually, volume flow at the aortic root $f_a(t)$ is not measured. Consequently, methods of approximating the model driving waveform $f_a(t)$ under practical measurement conditions are also investigated and discussed.**


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## INTRODUCTION

IN a recent paper, Clark *et al.* [1] developed an identification scheme for the determination of several parameters associated with a modified "windkessel" model of the systemic arterial system of an individual patient (Fig. 1). This identification scheme utilizes a modification of the Prony method [2], [3] as a "starter" method to determine good nominal values for the model parameters being varied. These values then serve as input to a well-known iterative nonlinear least-squares identification method (Marquardt method [4]) which converges rapidly to final values for the parameters. Numerical solution of the model differential equations with these "identified" parameter values yields the best fit in a least-squares sense of model-generated and observed central aortic root and distal brachial artery pressures. When coupled with a method for determining the contractile mechanics of the left ventricle (e.g., the ventricular elastance curve [5], [6]), this identification scheme permits a functional characterization of the left ventricle and its systemic load, for an individual subject.

The use of parameter estimation in the analysis of biological systems has grown rapidly in recent years [7]. However, Bekey and Grove [8] have recently pointed out the importance of sensitivity analysis in parameter estimation problems, particularly in biological systems where: (1) the system under study may involve many parameters whose values must be assumed